Physica A 507 (2018) 427–434

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Evolutionary of online social networks driven by pareto wealth distribution and bidirectional preferential attachment

Bin Zhou [a], Xiao-Yong Yan [b], Xiao-Ke Xu [c,*], Xiao-Ting Xu [a], Nianxin Wang [a]

[a] School of Economics and Management, Jiangsu University of Science and Technology, ZhenJiang, 212003, China
[b] Systems Science Institute, Beijing Jiaotong University, Beijing 100044, China
[c] College of Information and Communication Engineering, Dalian Minzu University, Dalian 116600, China

HIGHLIGHTS

- The origin of online social network evolution is analyzed and the evolution mechanisms are also explained.
- A model based on the Pareto Wealth Distribution and bidirectional preferential attachment is proposed.
- The evolutionary analysis of the proposed model is presented.
- The model can reproduce the essential evolution characteristics which are consistent with the ones of real-life online social networks.

ARTICLE INFO

Article history:
Received 8 February 2018
Received in revised form 19 April 2018

Keywords:
Online social network
Social stratification
Pareto distribution
Bidirectional preferential attachment

ABSTRACT

Understanding of the evolutionary mechanism of online social networks is greatly significant for the development of network science, but up to now most present researches on this topic have not enough insight. In this study, firstly we empirically showed the essential evolution characteristics of Renren online social network. The evolution mechanism of online social networks is explained by the perspective of Pareto wealth distribution and bidirectional preferential attachment. Then a novel model is proposed to reproduce the evolution characteristics which are consistent with the ones of Renren online social network, and the evolutionary analytical solution to the proposed model was presented. The results suggest that both Pareto wealth distribution and bidirectional preferential attachment play an important role in the evolution process of online social networks and can help us to understand the evolutionary origin of online social networks. The model has significant implications for dynamic simulation researches of social networks, especially in information diffusion through online communities and infection spreading in real societies.

With the rapid development of information technology, online social network platforms have appeared with a novel organizational form that differs from traditional social networks, in which the users maintain constant contacts and share the common interest, such as Facebook, Twitter, Renren, and Tecent QQ. Millions of people rely on online social networks to communicate with others, and their interactions generate new knowledge [1]. Thus the statistics and dynamics of online social networks are tremendously important to the researchers who are interested in human behaviors [2,3]. The systematic research on online social network data has created a new field of network sociology which integrates theories of traditional social networks and complex networks. Especially, network science has constituted a fundamental framework for analyzing and modeling online social networks [4].

* Corresponding author.
E-mail address: xuxiaoke@foxmail.com (X.-K. Xu).

https://doi.org/10.1016/j.physa.2018.05.049
0378-4371/

Traditionally, studies in the field of complex networks concentrated on the structural analysis of online social networks [5,6]. One of the most essential structural characteristic for online social networks is that the degree distribution follows a power-law instead of a normal distribution [7,8]. The degree of an individual is the number of friends that the individual has, and the degree distribution is the fraction for the individuals in the network who have exactly the number of friends. Among many different theories for generating a power-law distribution, the most prominent explanation is the Barabási and Albert model (BA model) of the preferential attachment [8]. The BA model is so well established that the preferential attachment is sometimes believed to be the origin of the power-law distribution. The community structure and the network connectivity are also important structure characteristics of online social networks, because they play an important role in information diffusion [9,10] and disease spread [11–15]. The community structure means that the nodes of the network can be easily grouped into sets of nodes such that each set of nodes is densely connected internally. A connected component is a set of individuals among which each pair of individuals are connected by at least one path through the network.

However, in this paper, our empirical analysis on Renren online social network indicates that the degree distribution is not a single power-law distribution but a two-region power-law distribution. The similar degree distributions of friend relationship have also been found in other online social networks [16]. The evolutionary characteristics of both the community structure and the network connectivity in Renren online social network are also not consistent with the ones in the simulation network generated by present models of social network [2,8,17,18]. Therefore, the state of art models cannot explain the essential structure characteristics of Renren online social network. A realistic online social network model needs to satisfy the essential structure characteristics of online social networks. Theoretical studies of dynamical processes and collective behavior taking place in online social networks would benefit from the realistic social network models [19].

The goal of this study is to analyze the evolutionary mechanisms of online social networks and propose a novel model to reproduce the essential multiple structure characteristics of Renren online social network. our results uncover that Pareto wealth distribution and bidirectional preferential attachment can play an important role in the evolution process of online social networks. This paper is structured as follows: firstly, we make the empirical demonstration of Renren online social network and show its essential structure characteristics. Secondly, we analyze the inherent features of online social networks and propose a novel social mechanism to explain the essential structure characteristics. Thirdly, a model is provided based on the proposed mechanism and the evolutionary analytical solution to the degree distribution is presented. Fourthly, our simulations reproduce the essential structure characteristics of Renren online social network. Furthermore, the model can also reproduce the rule of the common power-law degree distribution in complex networks. Finally, we discuss the significance of the work and conclude with a brief summary of our results.

1. Empirical demonstration

The evolution of complex systems in nature and society is from the initial unstable state to the final stable state. The mechanism of driving system evolving plays a dominant role in the evolution process from the initial unstable state to the final stable state. When the system reaches a stable state or dynamic equilibrium, the role of mechanism of driving system evolving will weaken or even disappear. Therefore, analyzing the evolution process from the initial unstable state to the final stable state will help us to explore deeply the mechanism of driving system evolution. The online social network is one of the most complex systems in nature and society. Although there are many online communications that form complex online social networks, detailed topological data is available for only a few, especially for the networks including the evolution process from the initial unstable state to the final stable state. The friend relationship graph of Renren Internet communication represents a well-documented example of the online social network, which is one of the largest online social networks in China. The website of Renren is http://www.renren.com. The online community is a dynamical evolving one with the new users joining in the community and new connections established between users. Each registered user of Renren has a profile, including his/her list of friends. If we view the users as nodes V and friend relationships as links E, an undirected friendship network $G(V, E)$ can be constructed from Renren. For privacy reasons, the data, logged from 21 November 2005 (the inception day for the Internet community) to 26 February 2006, include only each user's ID and list of friends, and the establishment time for each friend relationship.

Fig. 1(a)–(c) show chronologically the evolution process of degree distribution in Renren online community in the initial three months. The degree distribution changed from the initial single power-law distribution to the final two-region power-law distribution. The evolution characteristic indicates that some social mechanisms should play a leading role in the evolution process. Fig. 1(d) shows the evolution process of the average degree in the Renren online social network and the average number of friends for a user increases gradually. The community structure of Renren online social networks is detected using the Louvain method [20]. Fig. 1(e) shows the evolution process of community count and connected component count in the Renren online social network. The number of connected components first increases to the peak and then decreases. The evolution tendency of community count almost is consist with the one of the connected component count in the evolution process. We can further quantify this division using the modularity [21]. The modularity is the fraction of edges within communities minus the expected fraction of edges within communities in a randomized version of the network that preserves the degrees for each individual, but is otherwise random [22]. Fig. 1(f) shows that the evolution process of the modularity of the Renren online social network. The modularity decreases in the evolution process.

Take the prominent BA model as an example, Fig. 2(a)–(c) show the evolution process of the degree distribution of the simulation network generated by BA model. The degree distribution always follows the power-law. Due to the rules

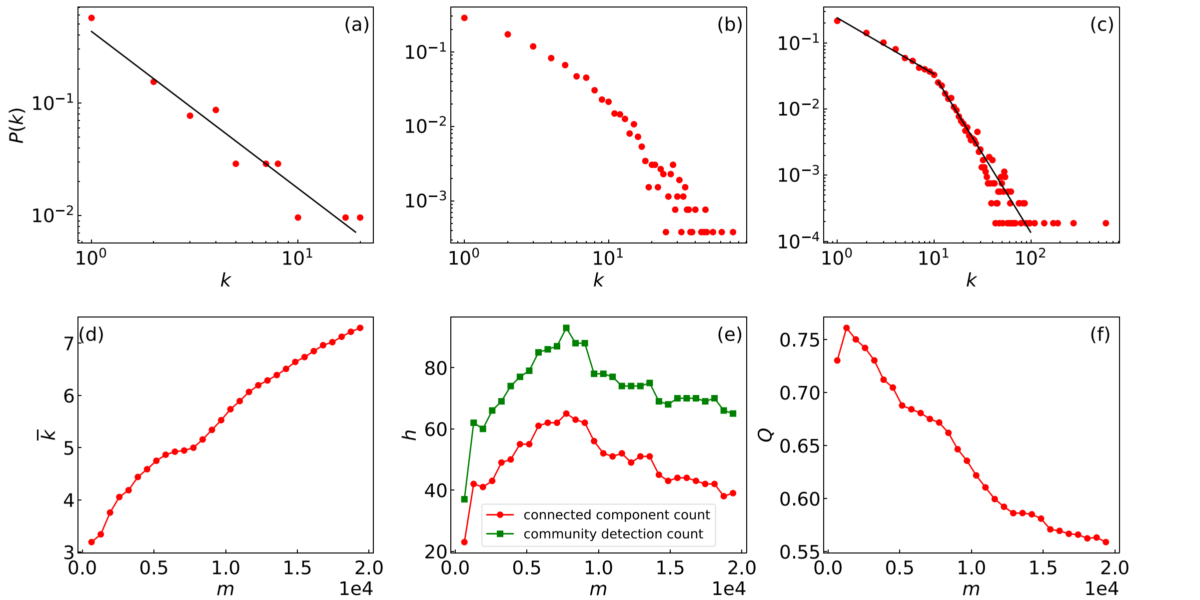

Fig. 1. The topological evolution of Renren online social network. Fig. 1(a)–(c) are three snapshots of degree distribution in the evolution process. In Fig. 1(a), the number of nodes is 104, and the number of links is 129, and the time is 22 December 2005. In Fig. 1(b), the number of nodes is 2625, and the number of links is 6460, and the time is 19 January 2006. In Fig. 1(c), the number of nodes is 5314, and the number of links is 19 318, and the time is 26 February 2006. In Fig. 1(a) and (c), the black solid lines represent the fitting results of the empirical data. Fig. 1(d)–(f) show respectively the evolution process of the average degree, community count, connected component count and modularity from 21 November 2005 to 26 February 2006.

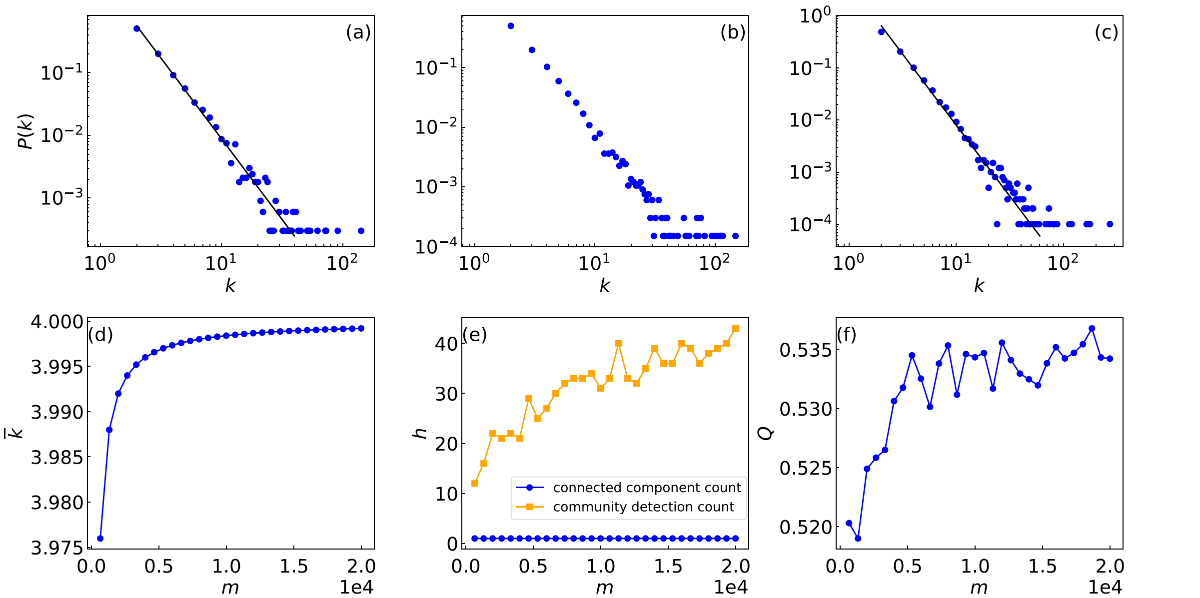

Fig. 2. The topological evolution of BA model. For BA model, a new node with two links is added in every time step and the evolution time is 10 000 time steps. Fig. 2(a)–(c) are three snapshots of degree distribution in the evolution process. In Fig. 2(a), the number of nodes is 3330, and the number of links is 6660, and the time step is 3330. In Fig. 2(b), the number of nodes is 6660, and the number of links is 13 320, and the time step is 6660. In Fig. 2(c), the number of nodes is 10 000, and the number of links is 20 000, and the time step is 10 000. In Fig. 2(a) and (c), the black solid lines represent the fitting results of the simulation data. Fig. 2(d)–(f) show respectively the evolution process of the average degree, community count, connected component count and modularity in the 10 000 time steps.

of BA model, the average degree is constant in the evolution process in Fig. 2(d). Fig. 2(e) shows the evolution process of the community count and the connected component count in BA model. The number of the connected component is one, which indicates that the whole simulation network is connected. The community count increases in the evolution process.

Fig. 2(f) shows that the modularity of the simulation network increases in the evolution process. By comparing Fig. 1(a)–(f) with Fig. 2(a)–(f) respectively, we find that the evolution tendencies of structure characteristics of the simulation network generated by BA model are not consistent with the ones of Renren online social network.

2. Empirical analysis

What basic interactions and linking mechanisms result in the essential evolution characteristics of Renren online social network? We will analyze the mechanisms and explain the essential evolution characteristics of Renren online social network. In the evolution process of BA model, they add a new node with m links that link the new node to m different nodes already present in the network at every time step. To incorporate preferential attachment, they assume the probability that a new node with m links will be connected to an old node depends on the connectivity of the old node. Aiming at online social networks, we think there are three generic aspects that are not incorporated in BA model.

First, according to the rule of BA model, the functional relation between the scale N of the simulation network and the evolution time t satisfies the function $N = m_0 + mt$. Therefore, the scale of the simulation network can grow indefinitely with the time evolution. In contrast, the scale of all online social networks cannot grow indefinitely with the time evolution, because the scale of human beings in real society is limited. For the same reason, the scale of Renren online social network is also impossible to grow indefinitely.

Second, they assume the probability that a new node with links will be connected to an old node depends on the connectivity of the old node in BA model. Therefore, the process of generating a link only involves the connectivity information of the old node and not involves other information of the old node. In online social networks, other information of the users may play an important role in the evolution process of online social networks. The essential evolution characteristics of Renren online social network may be caused by other information of users. The most important one in other information ignored is the social stratification of users in real society. Social stratification is a society's categorization of people into socioeconomic strata, based upon their occupation and income, wealth and social status, or derived power (social and political). As such, stratification is the relative social position of persons within a social group, category, geographic region, or social unit [23]. An individual hierarchy in social stratification has an important influence on the process of establishing his or her social circle. Social stratification of people should play a dominant role in the evolution process of online social networks. Strictly quantitative economic variables are extremely useful for describing social stratification. Wealth variables can vividly illustrate salient variations in the well-being of groups in stratified societies [24]. A lot of empirical researches have shown that individual wealth follows Pareto distribution [25–27]. The Pareto distribution, named after the Italian civil engineer, economist, and sociologist Vilfredo Pareto, is a power-law probability distribution that is used in description of social, scientific, geophysical, actuarial, and many other types of observable phenomena [28,29]. An individual hierarchy in Pareto wealth distribution has an important influence on the process of establishing his or her social circle. Therefore, Pareto wealth distribution may also have an important influence in the evolution process of online social networks.

Third, we need to analyze the rule of establishing friend relationship between any two users in the evolution process of online social networks. To incorporate preferential attachment in BA model, they assume the probability that a new node with m links will be connected to an old node depends on the connectivity of the old node. If the new node selected a node by the preferential attachment mechanism, the selected node must establish a connection with the new node. The preferential attachment mechanism of BA model only involves the demand of the new node and does not involve the demand of the selected node. Consequently, the process of establishing a connection between the new node and the selected node is a unidirectional selection process in BA model, and the preferential attachment mechanism of BA model is a unidirectional selection mechanism. However, in many cases, the unidirectional selection mechanism is unsuitable to explain the process of establishing friend relationship in human social behaviors, and many human social behaviors are a universal bidirectional selection process [30,31]. Take Renren as an example, when a user makes a request to another user and wants to establish friendship with another user, the requested user can choose to accept the request or can also choose to reject the request. Only when the requested user accepts the request, the friendship can be established between the two users. Therefore, the process of establishing friend relationship between two users is a bidirectional selection process. Considering the important influence of individual wealth on the evolution process of online social networks, the process of establishing friend relationship between two users should be extremely relevant with the wealth information of the two users. We assume that each user wants to preferentially establish friend relationship with other users that have more wealth, so the friend relationship is easier to be established among those users with more wealth. Compared with the unidirectional referential attachment mechanism of BA model, our bidirectional selection mechanism of establishing friend relationship between two users is a bidirectional preferential attachment mechanism.

3. Model

We build a model based on the blend mechanisms above, which can reproduce the observed essential evolution characteristics of Renren online social network. To incorporate the limited scale of simulation social network, we assume there are all N independent individuals, i, j and l are the No. of an individual, and the reasonable ranges of i, j and l are all from 1 to N. The upper limit of the scale of the simulation social network is also N. To incorporate the power-law distribution

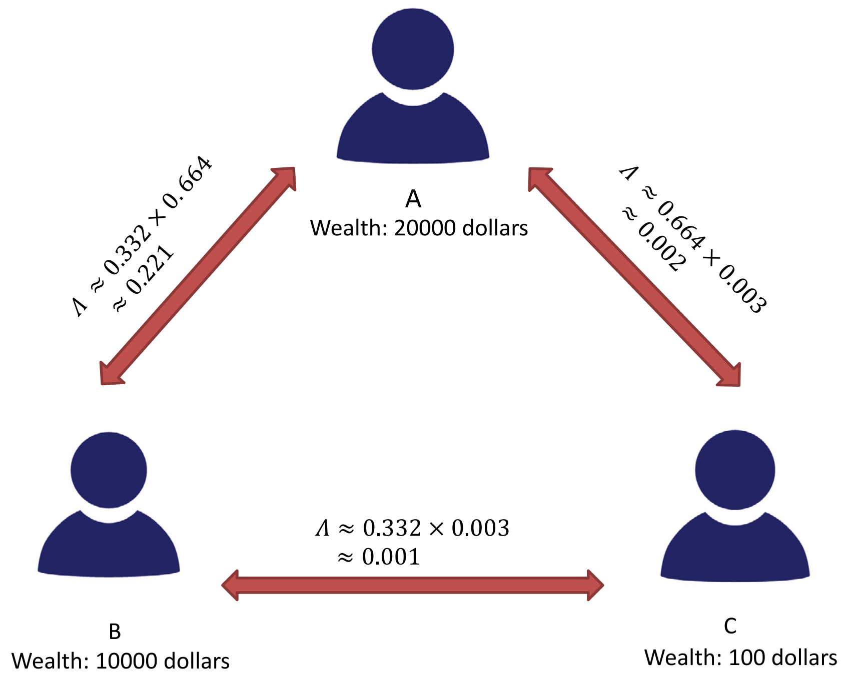

Fig. 3. The illustration of the proposed model. Λ is the probability that the fiend relationship is established between two individuals.

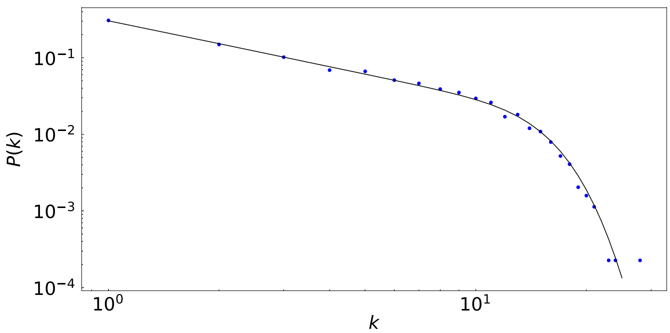

Fig. 4. Comparison between the simulation results and the analysis results. The parameters are $N = 10\,000$, $t = 10\,000$, $a = 1$, $b = 1000$, $\alpha = 1$. The blue circle and the black solid line represent the simulation results and the analysis results, respectively.

of individual wealth, each individual is assigned a wealth value ω according to the power-law distribution $p(\omega) = c \times \omega^{-\alpha}$, where c is the normalized constant, and ω is the positive integral $\omega \in [a, b]$, α is the power exponent, a, b are both positive integers. To incorporate the bidirectional preferential attachment mechanism, we assume that the probability Λ that an individual i will be chosen depends on the wealth ω_i of the individual, so that $\Lambda(\omega_i) = \omega_i / \sum_{l=1}^{N} \omega_l$. At every time step, two individuals are chosen independently to establish friend relationship and are connected by a link, and the probability that the two individuals i and j are chosen independently and are connected by a link depends on the wealth ω_i and ω_j of the two individuals, so that $\Lambda(\omega_i, \omega_j) = \omega_i \times \omega_j / (\sum_{l=1}^{N} \omega_l \times \sum_{l=1}^{N} \omega_l)$. Therefore, there is a higher probability that the friend relationship will be established between two individuals with more wealth. After t time steps of evolution, the model can generate a simulated social network.

The illustration Fig. 3 can help us to well understand the evolution rule of the model. There are three individuals in a social circle, signed as A, B and C. The wealth values of the three individuals is 20 000, 10 000 and 100 dollars, respectively. The total wealth for them is 30 100 dollars. The wealth proportion of A, B and C in total wealth is about 0.664, 0.332 and 0.003, respectively. At every time step, two individuals are chosen from the social circle to establish friend relationship. The probability that A and B are chosen to establish friend relationship is about 0.221(0.664 × 0.332), and the probability that A and C are chosen to establish friendship is about 0.002(0.664 × 0.003), and the probability that B and C are chosen to establish friendship is about 0.001(0.332 × 0.003). It only needs less than five time steps (1/0.221) of evolution that the friend relationship can be established between A and B, but it needs about five hundred or one thousand time steps (1/0.002

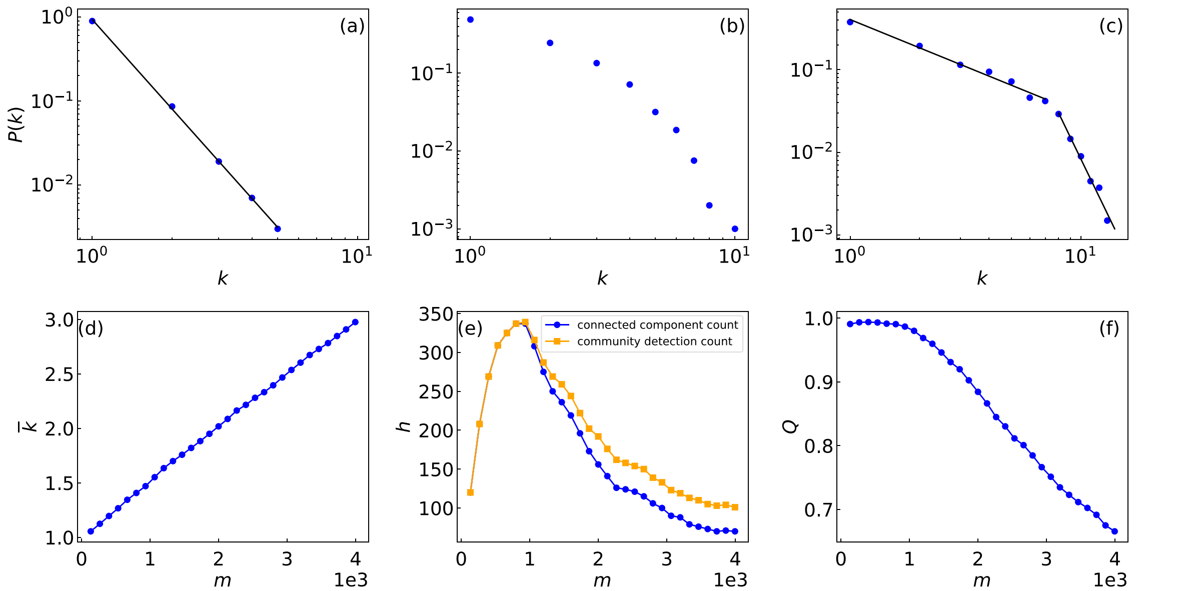

Fig. 5. The topological evolution of the simulation network generated by the model. The parameters of the model are $N = 10\,000$, $t = 4000$, $a = 1$, $b = 1000$, $\alpha = 1$. Fig. 5(a)–(c) are three snapshots of degree distribution in the evolution process. In Fig. 5(a), the number of nodes is 475, and the number of links is 267, and the time step is 267. In Fig. 5(b), the number of nodes is 1952, and the number of links is 2000, and the time step is 2000. In Fig. 5(c), the number of nodes is 2686, and the number of links is 4000, and the time step is 4000. In Fig. 5(a) and (c), the black solid lines represent the fitting results of the simulation data. Fig. 5(d)–(f) show respectively the evolution process of the average degree, community count, connected component count and modularity in the 4000 time steps.

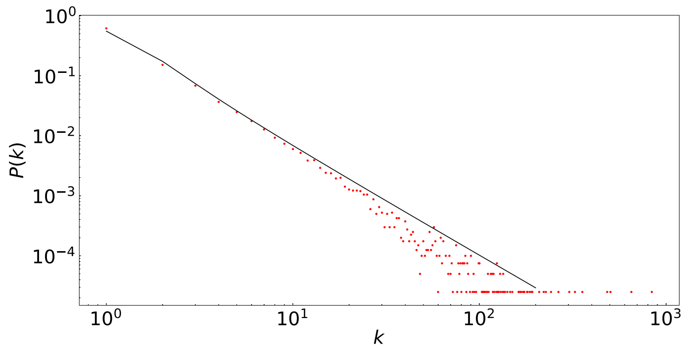

Fig. 6. Comparison between the analysis results and the experimental results of Flickr. The red circle and the black solid line represent the experimental results and the analysis results, respectively. The numbers of nodes and links in Flickr are 40 108 and 62 828, respectively. The parameters of the analytical solution are $N = 40\,108$, $t = 62\,828$, $a = 1$, $b = 1000$, $\alpha = 1.8$.

or 1/0.001) of evolution that C can only establish friend relationship with A or B. Apparently, the probability that the friend relationship is established between A and B is far higher than the one between A and C or the one between B and C.

4. Model analysis and simulation

We will derive the analytical solution of the degree distribution evolution in the simulation network as follows. In the model, each individual is assigned a wealth value ω according to the power-law distribution $p(\omega) = c \times \omega^{-\alpha}$, $\omega \in [a, b]$, and the average wealth ϖ of an individual is $\varpi = \sum_{\omega=a}^{b} \omega \times c \times \omega^{-\alpha}$. The probability that an individual with the wealth hierarchy ω is chosen is $\Lambda(\omega) = \omega/(N \times \varpi)$ At every time step, two individuals are chosen independently to create one pair of friends and are connected by a link, and the probability that two individuals i and j are chosen independently and are connected by a link is $\Lambda(\omega_i, \omega_j) = \omega_i \times \omega_j/(N \times \varpi)^2$. After t time steps of evolution, a simulated social network is generated. In most of real online social networks, their scale is great large and the average degree of an individual is much smaller than the scale. Therefore, most online social networks are sparse networks. In the model, when N is great large and $t \ll N^2$, the simulated social network is a sparse network. Both the probabilities of two different individuals being chosen more than once in t time steps and one individual being chosen twice at one time step are almost zero. Consequently, the duplicate links and self-connected links in simulated social network can be ignored. At one time step, once an individual is chosen, the degree of the individual is increased by one. In t time steps of evolution, the combination count of an individual being chosen k times is $t!/(k! \times (t-k)!)$. For each combination, the probability that an individual with the wealth hierarchy ω is chosen k times is $(2 \times \omega/(N \times \overline{\omega}))^k$. In the rest of $t-k$ time steps, the probability that an individual with the wealth hierarchy ω is not chosen is $(1 - 2 \times \omega/(N \times \overline{\omega}))^{t-k}$. Considering the wealth hierarchy ω of an individual is assigned according to the power-law distribution $p(\omega) = c \times \omega^{-\alpha}$ and $\omega \in [a, b]$, the probability $P(k)$ that an individual has k friends after t time steps of evolution is as follows

$$P(k) \approx \sum_{\omega=a}^{b} c \times \omega^{-\alpha} \times \binom{t}{k} \times \left(\frac{2 \times \omega}{N \times \overline{\omega}}\right)^{k} \times \left(1 - \frac{2 \times \omega}{N \times \overline{\omega}}\right)^{t-k}, \qquad t \ll N^2. \tag{1}$$

Fig. 4 shows the comparison of the degree distribution between the analysis results and the simulation results. The analysis results are in good agreement with the simulation results, which indicates that the analytical results are reliable.

Fig. 5 shows the topological evolution of the simulation network generated by the model. Fig. 5(a)–(c) show chronologically the evolution process of the degree distribution. The degree distribution changed from the initial single power-law distribution to the final two-region power-law distribution. Fig. 5(d) shows the evolution process of the average degree and the average number of friends for an individual increase gradually. Fig. 5(e) shows the evolution processes of the community count and the connected component count. The connected components count first increases to the peak and then decreases. The evolution tendency of the community count almost is consist with the one of the connected component count in the evolution process. Fig. 5(f) shows that the modularity decreases in the evolution process. By comparing Fig. 1(a)–(f) with Fig. 5(a)–(f) respectively, we find that the evolution tendencies of the structure characteristics of the simulation network are consistent with the ones of the structure characteristics of Renren online social network. Therefore, a blended mechanism including the limited network scale, the power-law distribution of individual wealth and the bidirectional preferential attachment can reproduce the essential evolution characteristics of Renren online social network. The topology structure of the simulation network is similar to the one of Renren online social network. The model can help us to understand the evolutionary origin of online social networks from the perspective of Pareto wealth distribution and bidirectional preferential attachment.

In addition, the model can well reproduce the ordinary power-law distribution of online social networks [7,8]. Fig. 6 shows the comparison of the degree distribution between the analysis results and the experimental results of Flickr. Flickr is an online social network, which is from public data platform [32], and the website is http://socialnetworks.mpi-sws.org/data-wosn2008.html, and the degree distribution in Flickr follows power-law. The numbers of nodes and links in Flickr are substituted into the analytical solution, and the analysis results of the degree distribution are in favorable agree with the empirical results of the degree distribution.

5. Conclusion

Firstly, we empirically show the essential evolution characteristics of Renren online social network. From the perspective of Pareto wealth distribution and bidirectional preferential attachment, the evolutionary origin of online social networks is analyzed and explained. Then a model is proposed based on the limited network scale, Pareto wealth distribution and bidirectional preferential attachment mechanism. Furthermore the analytical solution of the model is provided. The simulation results of the model indicate that the model can reproduce the essential evolution characteristics of Renren online social network. Secondly, our model can also reproduce the single power-law degree distribution. Therefore, Pareto wealth distribution and bidirectional preferential attachment can play a dominant role in the evolution process of online social networks, leading from simple individuals to complex online social networks. At last, the model can help us immensely to understand evolutionary origin of online social networks and has great significance in dynamic simulation of online social networks.

Because the topological structure of the simulation social network is similar to the one of real-life online social network, the model of the blended mechanisms has important implications for the dynamic simulation researches of online social networks. The evolutionary characteristics of both the community and the connected component in the simulation social network are in consistent with the ones in Renren online social networks. These properties play an important role in both information diffusion and infection spreading. The model not only can simulate the two-region power-law degree distribution but can also reproduce the single power-law degree distribution. Therefore, the blended mechanisms of the model may be a generic property to online social networks and uncover the evolutionary origin of online social networks. The social stratification and the bidirectional preferential attachment also may be common mechanisms to others complex networks, such as business networks, biological networks and engineering networks. We expect that the evolutionary mechanisms of online social networks can be expanded to other fields of complex networks and help to understand the evolutionary origin of other complex systems, with applicability reaching far beyond the quoted examples.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant Nos. 61503159, 71331003, 71471079, 61773091, 61603073) and the Jiangsu philosophy and Social Science Foundation, China (Grant No. 2016SJB630097).

References

[1] S. Faraj, S.L. Jarvenpaa, A. Majchrzak, Knowledge collaboration in online communities, Org. Sci. 22 (5) (2011) 1224–1239.
[2] P. Klimek, S. Thurner, Triadic closure dynamics drives scaling-laws in social multiplex networks, New J. Phys. 15 (6) (2013) 2012–2019.
[3] E. Ubaldi, N. Perra, M. Karsai, A. Vezzani, R. Burioni, A. Vespignani, Asymptotic theory of time-varying social networks with heterogeneous activity and tie allocation, Sci. Rep. 6 (2016) 35724.
[4] M. Newman, A.L. Barabási, D.J. Watts, The Structure and Dynamics of Networks, Princeton University Press, 2011, pp. 206–207.
[5] M.E.J. Newman, A. Clauset, Structure and inference in annotated networks, Nature Commun. 7 (2–3) (2016) 11863.
[6] K. Lewis, J. Kaufman, M. Gonzalez, A. Wimmer, N. Christakis, Tastes, ties, and time: A new social network dataset using Facebook.com, Social Networks 30 (4) (2008) 330–342.
[7] M.E.J. Newman, The structure and function of complex networks, Siam Review 45 (2) (2003) 167–256.
[8] A.L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (5439) (1999) 509–512.
[9] R. Lambiotte, P. Panzarasa, Communities, knowledge creation, and information diffusion, J. Inform. 3 (3) (2009) 180–190.
[10] A. Nematzadeh, E. Ferrara, A. Flammini, Y.Y. Ahn, Optimal network modularity for information diffusion, Phys. Rev. Lett. 113 (8) (2014) 088701.
[11] M. Salathé, J.H. Jones, Dynamics and control of diseases in networks with community structure, PLos Comput. Biol. 6 (4) (2010) 387–395.
[12] W. Wang, M. Tang, H.E. Stanley, L.A. Braunstein, Unification of theoretical approaches for epidemic spreading on complex networks, Rep. Progr. Phys. 80 (3) (2017) 036603.
[13] W. Wang, H.E. Stanley, L.A. Braunstein, Effects of time-delays in the dynamics of social contagions, New J. Phys. 20 (1) (2018) 013034.
[14] Q.-H. Liu, L.-F. Zhong, W. Wang, T. Zhou, H. Eugene Stanley, Interactive social contagions and co-infections on complex networks, Chaos 28 (1) (2018) 013120.
[15] P. Shu, L. Gao, P. Zhao, W. Wang, H.E. Stanley, Social contagions on interdependent lattice networks, Sci. Rep. 7 (2017) 44669.
[16] J. Ugander, B. Karrer, L. Backstrom, C. Marlow, The anatomy of the facebook social graph, Comput. Sci. (2011).
[17] J.M. Pacheco, A. Traulsen, M.A. Nowak, Coevolution of strategy and structure in complex networks with dynamical linking, Phys. Rev. Lett. 97 (25) (2006) 258103.
[18] R. Toivonen, J.P. Onnela, J. Saramäki, J. Hyvönen, K. Kaski, A model for social networks, Physica A 371 (2) (2006) 851–860.
[19] L. L, D.B. Chen, T. Zhou, Small world yields the most effective information spreading, New J. Phys. (12) (2012) 825–834. arXiv:abs/1107.0429.
[20] V.D. Blondel, J.L. Guillaume, R. Lambiotte, E. Lefebvre, Fast unfolding of communities in large networks, J. Stat. Mech. Theory Exp. 2008 (10) (2008) 155–168.
[21] M.E. Newman, M. Girvan, Finding and evaluating community structure in networks, Phys. Rev. E 69 (2) (2004) 026113.
[22] M. Molloy, B. Reed, A critical point for random graphs with a given degree sequence, Random Struct. Algorithms 6 (2–3) (1995) 161–180.
[23] P. Saunders, Social Class and Stratification, Routledge, 2006, pp. 61–69.
[24] J. Angle, The surplus theory of social stratification and the size distribution of personal wealth, Soc. Forces 65 (2) (1986) 293–326.
[25] P. Vermeulen, How fat is the top tail of the wealth distribution?, Rev. Income Wealth (2017).
[26] M. Levy, S. Solomon, New evidence for the power-law distribution of wealth, Physica A 242 (1–2) (1997) 90–94.
[27] O.S. Klass, O. Biham, M. Levy, O. Malcai, S. Solomon, The forbes 400 and the pareto wealth distribution, Econom. Lett. 90 (2) (2006) 290–295.
[28] V. Pareto, G.H. Bousquet, G. Busino, Cours d'économie politique, Rev. Écon. 16 (5) (2015) 811.
[29] C.B. Arnold, Pareto Distribution, John Wiley and Sons, Ltd., 2015.
[30] B. Zhou, Z. He, L.L. Jiang, N.X. Wang, B.H. Wang, Bidirectional selection between two classes in complex social networks, Sci. Rep. 4 (2014) 7577.
[31] B. Zhou, S. Qin, X.P. Han, Z. He, J.R. Xie, B.H. Wang, A model of two-way selection system for human behavior, Plos One 9 (1) (2014) e81424.
[32] A. Mislove, H.S. Koppula, K.P. Gummadi, P. Druschel, B. Bhattacharjee, Growth of the flickr social network, in: Proceedings of the First Workshop on Online Social Networks, 2008, pp. 25–30.