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A model of task-deletion mechanism based on the priority queueing system of Barabási



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HIGHLIGHTS

- A task-deletion mechanism is added to the Barabási decision model.
- A decision model of task-deletion mechanism is proposed.
- The model can produce rich statistical properties of human behavior patterns.
- These results are helpful for understanding the mechanism of pattern diversity.
- Our work is helpful for understanding the source of pattern diversity of human behaviors.

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ABSTRACT

In this paper, we propose a model of task-deletion mechanism based on the priority queueing system of Barabási (2005) to deep research the pattern diversity of human behaviors. The analytical solution for our model with two tasks is presented. In different cases of the parameter of task-deletion, our model can produce rich statistical behavior patterns, which are consistent with lots of empirical studies. Therefore, the model can theoretically explain more human behavior phenomena than the model of Barabási. These results have important significance for understanding the mechanism of pattern diversity of human behaviors.

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1. Introduction

Humans involve in a daily basis in lots of distinct behaviors, ranging from electronic communication (such as sending e-mails or making telephone calls) to browsing the Internet, initiating financial transactions, or engaging in entertainment and sports. The individual human behaviors drive the dynamics of many social, technological and economic phenomena, and the dynamics turn the quantitative understanding of human behavior into a central question of modern science. The emergence of several problems inspired our interest to understand human behavior patterns [1–3]. At first, human behaviors are assumed in time that the execution of each task is independent from the others and is executed at a constant rate [4]. Therefore, the processes of executing tasks can be well modeled by Poisson progresses [1–4], such as sending e-mail or making telephone calls. The time interval between two consecutive executions of a task by the same individual, called

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the waiting time, follows an exponential distribution. Later, with the deeper research of human behavior, an increasing evidence indicates that the timing of massive human behaviors follow non-Poisson statistics, characterized by bursts of rapidly occurring events separated by long periods of inactivity [5-8]. The waiting time of a task executed exhibits a heavy tail and follows the power-law distribution. Barabási proposed a priority queueing system to model the processes in the human behavior, where the human plays role of the server [9].

The Barabási model of human processes well describes the pattern of heavy tail. In the extremal dynamics limit, when an individual selects the highest priority task first, numerical simulations and heuristic arguments show that most of the tasks are executed in one time step, while the waiting time distribution of tasks waiting more than one time step exhibits a heavy tail and follows the power-law distribution in the processes. The exact results of waiting time distribution for the model of Barabási with two tasks are obtained [10]. The model of Barabási is great helpful for us to understand the origin of the power-law distribution in the human behaviors, such as the emergence of power-laws in online communities [11]. These findings have important implications, ranging from resource management to service allocation, in both communications and retail.

In addition to the widespread exponential distribution and the power-law distribution in real world, further lots of experimental researches showed that the human behavior patterns are neither completely Poisson nor power-law but are a mixing distribution of power-function and exponential-function [12–16]. In particular, human behavior patterns probably are power-law distribution followed by distinct cutoff [17–21], e.g., exponential cutoff [22–30]. Such diverse statistical patterns cannot be explained completely by the Barabási model. How to explain the pattern diversity of human behaviors, and whether or not there exists a source which results in the pattern diversity of human behaviors. All these questions need to be researched deeply.

We try to explore the source which causes the pattern diversity of human behaviors as follows. In the Barabási model [9], an individual keeps track of a list with many active tasks that he or she needs to execute, and each task is assigned a priority. The decision that the task with the highest priority will be executed first results in the power-law distribution of the waiting time. In most human-initiated behaviors, the individual indeed executes the task with the highest priority first. But time is limited and precious resource for each individual, the individual probably does not execute all tasks. When the priority of a task is lower than a certain threshold, the individual will make decision to delete it. For instance, in modern society, most of people have these experiences that they often receive e-mail or short message of mobile. If they think the e-mail or the short message is important, they will reply it. If not, they will not reply it and delete it from the mailbox or the mobile phone. The thresholds also are probably different among different individuals. We conjecture that the task-deletion mechanism is probably the source which causes the pattern diversity of human behaviors.

In this paper, a task-deletion mechanism is added to the priority queueing system of the Barabási model to deeper research the pattern diversity of human behaviors. Firstly, the model of task-deletion mechanism is described in detail. We calculate the priority and waiting time distribution with two tasks in the processes of executing tasks. The exact results are obtained for the model of task-deletion mechanism. Secondly, the simulation results of our model are done and we make an in-depth discussion for the simulation results. Thirdly, we summarize our work and the significances of our work are explored. Finally, our thanks are expressed.

2. The model and analytical results

Our model of task-deletion mechanism is stated as follows:

- (i) An individual has a list with *L* active tasks that he or she needs to execute. Each active task is assigned a priority $x (0 \le x \le 1)$ from a probability density function $\rho(x)$.
- (ii) At each time step, the individual selects the task with the highest priority from the list with probability p, and with probability 1 p he or she randomly selects a task, independent of its priority.
- (iii) There is a certain threshold of deleting task, called μ ($0 \le \mu \le 1$). If the priority of the selected task is lower than a certain threshold μ , the selected task is not executed and deleted from the list, and the waiting time of the selected task is also not recorded. If not, the selected task is executed, removed from the list, and the waiting time of the selected task is recorded. Whether or not the selected task is executed, at that moment a new task is added to the list, its priority being assigned from $\rho(x)$.

The numerical simulations show that the case L = 2 already exhibits the relevant features of the Barabási [9]. Therefore, we next consider our model with two tasks and obtain the exact results of our model, calculating the priority and waiting time distribution. At each time step, the selected task which is reassigned a priority *x* from $\rho(x)$ will be called the new task, and the other task will be called the old task. In the stationary state, the priority probability functions of the new task and the old task are independent of time. $\rho(x)$ and $R(x) = \int_0^x \rho(x') dx'$ are the priority probability density function and the distribution function of the new task, respectively. $\rho_1(x)$ and $R_1(x) = \int_0^x \rho_1(x') dx'$ are the ones of the old task, respectively. If the old task has priority *x*, the probability that the new task is selected follows that

$$q(x) = p[1 - R(x)] + (1 - p)\frac{1}{2}.$$
(1)

If the new task has priority x, the probability that the old task is selected follows that

$$q_1(x) = p[1 - R_1(x)] + (1 - p)\frac{1}{2}.$$
(2)

After one task being selected, the old task will have the distribution function

$$R_1(x) = \int_0^x \rho_1(x')q(x')\,\mathrm{d}x' + \int_0^x \rho(x')q_1(x')\,\mathrm{d}x'. \tag{3}$$

Using Eqs. (1)–(2) and integrating equation (3) we obtain

$$R_1(x) = \frac{(1+p)R(x)}{1-p+2pR(x)}.$$
(4)

In the stationary state, we calculate the priority and the waiting time distribution of executed tasks. At some time step, a task with priority x ($x \ge \mu$) has just been added to the queue. The probability that it waits τ time steps to be executed is given by the product of the probability that it is not selected in the first $\tau - 1$ time steps and that it is selected in the τ th time step. The probability that it is not selected in the first time step is $q_1(x)$, and the probability that it is not selected in the subsequent $\tau - 2$ time steps is q(x), and the probability that it is selected in the τ th time step is 1 - q(x). It should be mentioned that the task-deletion mechanism is added to the Barabási model. If the priority of the selected task is lower than a certain threshold μ ($0 \le \mu \le 1$), the selected task is not executed, deleted from the list, and the waiting time of the selected task is also not recorded. Simultaneously a new task is added to the list, its priority being reassigned from $\rho(x)$. Therefore, the individual has selected n ($n \ge \tau$) tasks in the process of the task waiting τ time steps to be executed. Among the *n* selected tasks, only the priorities of τ tasks are higher than or equal to the threshold μ , and the priorities of other $n - \tau$ tasks are lower than the threshold μ . Based on the above analysis, the waiting time distribution of executed tasks with the priority x ($x \ge \mu$) can be obtained as follows

$$P(\tau) = \begin{cases} \int_{\mu}^{1} \left\{ 1 - q_{1}(x) + \sum_{n=2}^{\infty} f_{1}(\mu) f(\mu)^{n-2} \left[1 - q(x) \right] \right\} \frac{\rho(x)}{1 - \mu} \, \mathrm{d}x, & \tau = 1 \\ \int_{\mu}^{1} \left\{ f_{1}(\mu) \sum_{n=\tau+1}^{\infty} C_{n-2}^{\tau-1} g(x)^{\tau-1} f(\mu)^{n-\tau-1} \left[1 - q(x) \right] \right. \\ \left. + g_{1}(x) \sum_{n=\tau}^{\infty} C_{n-2}^{\tau-2} g(x)^{\tau-2} f(\mu)^{n-\tau} \left[1 - q(x) \right] \right\} \frac{\rho(x)}{1 - \mu} \, \mathrm{d}x, & \tau \ge 2, \end{cases}$$
(5)

where f(x), $f_1(x)$, g(x) and $g_1(x)$ follow that

$$f(x) = R(x)\frac{1}{2}(1-p), \qquad f_1(x) = R_1(x)\frac{1}{2}(1-p),$$
(6)

$$g(x) = [R(x) - R(\mu)] \frac{1}{2}(1-p) + [R(1) - R(x)] \left[p + \frac{1}{2}(1-p) \right],$$
(7)

$$g_1(x) = [R_1(x) - R_1(\mu)] \frac{1}{2} (1-p) + [R_1(1) - R_1(x)] \left[p + \frac{1}{2} (1-p) \right].$$
(8)

We assume that the priority of each task is assigned from a uniform distribution $\rho(x) = 1$. It can be gotten that

$$R(x) = x, R_1(x) = \frac{(1+p)x}{1-p+2px}.$$
(9)

Using Eqs. (1)-(2), Eqs. (6)-(9) and integrating equation (5) we finally obtain

$$P(\tau) = \begin{cases} 1 - \frac{1 - p^2}{4p(1 - \mu)} \ln \frac{1 + p}{1 - p + 2p\mu} + \frac{R_1(\mu)(1 - p)(1 + p\mu)}{4 - 2\mu(1 - p)}, & \tau = 1\\ \frac{R_1(\mu)(1 - p)}{2(1 - \mu)\gamma^{\tau}p} \left(\frac{\alpha^{\tau+1} - \beta^{\tau+1}}{\tau + 1} - \gamma \frac{\alpha^{\tau} - \beta^{\tau}}{\tau}\right) \\ + \frac{1 - p}{2p(1 - \mu)\gamma^{\tau-1}} \left(\left(R_1(\mu)\gamma - \frac{1}{2}(1 + p) \right) \frac{\alpha^{\tau-1} - \beta^{\tau-1}}{\tau - 1} - R_1(\mu) \frac{\alpha^{\tau} - \beta^{\tau}}{\tau} \right), \quad \tau \ge 2, \end{cases}$$
(10)

where α , β and γ are as follows

$$\alpha = \frac{1}{2}(1-p)(1-\mu), \qquad \beta = \frac{1}{2}(1+p)(1-\mu), \qquad \gamma = 1 - \frac{1}{2}\mu(1-p). \tag{11}$$

The waiting time distribution with two tasks is a mixing distribution of power-function and exponential-function.

In one case of $\mu = 0$, our model returns to the Barabási mode [9], and from Eq. (10) it follows that

$$\lim_{\mu=0} P(\tau) = \begin{cases} 1 - \frac{1-p^2}{4p} \ln \frac{1+p}{1-p}, & \tau = 1\\ \frac{1-p^2}{4p} \left[\left(\frac{1+p}{2}\right)^{\tau-1} - \left(\frac{1-p}{2}\right)^{\tau-1} \right] \frac{1}{\tau-1}, & \tau \ge 2. \end{cases}$$
(12)

The analysis results are in agreement with the exact results of Vázquez [10]. When $p \rightarrow 0$ from Eq. (12) we obtain

$$\lim_{\substack{\mu=0\\p\to 0}} P(\tau) = \left(\frac{1}{2}\right)^{\tau}, \quad \tau \ge 1.$$
(13)

Under the condition of random selection protocol, the waiting time distribution of active tasks follows an exponential distribution. When $p \rightarrow 1$ from Eq. (12) we obtain

$$\lim_{\substack{\mu=0\\p\to 1}} P(\tau) = \begin{cases} 1 + \frac{1-p}{2} \ln(1-p), & \tau = 1\\ \frac{1-p}{2} \frac{1}{\tau-1}, & \tau \ge 2, \end{cases}$$
(14)

when $\tau \to \infty$, Eq. (14) results in

$$\lim_{\substack{\mu=0\\p>1\\\tau\to\infty}} P(\tau) = \frac{1-p}{2}\tau^{-1}.$$
(15)

Under the condition of highest priority first selection protocol, the waiting time distribution follows a power-law distribution for $\tau \to \infty$.

In the other case of $\mu \neq 0$, when $p \rightarrow 0$ from Eq. (10) we obtain

$$\lim_{p \to 0} P(\tau) = \frac{(1-\mu)^{\tau-1}}{(2-\mu)^{\tau}}, \quad \tau \ge 1.$$
(16)

The limit corresponds with the random selection protocol, where a task is selected with probability 1/2 on each step time, and the waiting time distribution follows an exponential distribution and is relevant to the deletion parameter μ . When $p \rightarrow 1$ from Eq. (10) we obtain

$$\lim_{p \to 1} P(\tau) = \begin{cases} 1 - \frac{1-p}{2(1-\mu)} \ln \frac{2}{1-p+2p\mu} + \frac{1}{4} R_1(\mu)(1-p)(1+\mu), & \tau = 1\\ \frac{R_1(\mu)(1-p)}{2(1-\mu)} \left(\frac{(1-\mu)^{\tau}}{\tau} - \frac{(1-\mu)^{\tau+1}}{\tau+1} \right) \\ + \frac{1-p}{2(1-\mu)} \left(R_1(\mu) \frac{(1-\mu)^{\tau}}{\tau} + (1-R_1(\mu)) \frac{(1-\mu)^{\tau-1}}{\tau-1} \right), & \tau \ge 2, \end{cases}$$
(17)

when $\tau \to \infty$, Eq. (17) results in

$$\lim_{\substack{p \to 1 \\ \tau \to \infty}} P(\tau) = \frac{1-p}{2} \tau^{-1} (1-\mu)^{\tau}.$$
(18)

The limit corresponds with the highest priority first selection protocol, where a task with the highest priority is selected, and the probability distribution $P(\tau)$ follows a power-law distribution with an exponential cutoff for $\tau \to \infty$.

In different cases of the task-deletion parameter, the analytical results above show that our model can produce rich statistical properties, such as mixing distribution of exponential-function and power-function, exponential distribution, power-law distribution and power-law distribution with an exponential cutoff, which are consistent with lots of empirical studies. These indicate that the method of adding the task-deletion mechanism to the Barabási model is reasonable. The model of task-deletion can theoretically explain the pattern diversity of human behaviors. It verifies our conjecture that the task-deletion mechanism is probably the source which causes the pattern diversity of human behaviors.



Fig. 1. (Color online) The simulation results of the model of task-deletion mechanism between *P* and τ in the log–log plot. The parameters *L* = 2, *p* = 0.999999. The light-blue dots, red squares, green diamonds, blue up triangles and black down triangles represent the corresponding results in the cases of $\mu = 0.00, 0.01, 0.10, 0.50$ and 0.90, respectively.



Fig. 2. (Color online) The analytical results from (10) in the log–log plot. The parameter p = 0.99999; μ is assigned to values from the interval from 0 to 1 and τ is assigned to values from 2 to 1000.



Fig. 3. (Color online) The comparison between the analytical results and the simulation results in the log–log plots. The parameters p = 0.99999 and $\mu = 0.01$. The black solid line is the analytical result from Eq. (18) and the red point line is the simulation result; τ is assigned to values from 2 to 1000.

3. Simulation results

In the different cases of the parameter μ , Fig. 1 shows the simulation results of the model of task-deletion mechanism between *P* and τ under the decision of selecting the task with the highest priority first. When $\mu = 0$, the waiting time distribution of the tasks that are not selected in the first step has a power-law tail and follows a power-law probability distribution, and these results can be verified by the analysis solution equation (14). When $\mu = 0.01$, the probability *P*(τ) follows a mixing distribution of power-function and exponent-function, and this results can be verified by the analysis solution equation (17). Therefore, the pattern of probability distribution *P*(τ) is highly sensitive to the parameter μ .

Fig. 2 displays the analytical results of Eq. (10) in the log–log plot. The deletion variable μ is various from 0 to 1. With the increasing of μ , the relations between *P* and τ changed from the linear power-law distribution to the curvilinear power-law distribution with an exponential cutoff. Because the parameter p = 0.99999, these changes can be verified by the limit-analysis solutions Eqs. (15) and (18), respectively. Under the condition of $\mu = 1$, the *P* decreased quickly with the increasing of τ , and when $\tau \ge 200$, *P* is lower than 10^{-300} , which indicates that P is highly sensitive to τ and the long waiting time τ is almost never appear. Fig. 3 shows the comparison between the analysis results of Eq. (18) and the simulation



Fig. 4. (Color online) The comparison between the two analytical results from Eq. (10) under the two conditions $\mu = 0$ and $\mu = 0.5$ in the log–log plots. In the two sub-figures, the parameters *p* is assigned to values from 0 to 1 and τ is assigned to values from 2 to 1000. (a) and (b) are corresponding to the two conditions $\mu = 0$ and $\mu = 0.5$, respectively.

results. The analytical results and the simulation results are consistent with each other. That is to say the limit-analysis solution equation (18) is reliable.

Fig. 4 shows the comparison between the two analytical results of Eq. (10) under two conditions $\mu = 0$ and $\mu = 0.5$. Fig. 4(a) and (b) are corresponding to the two conditions $\mu = 0$ and $\mu = 0.5$, respectively. The variable *p* is various from 0 to 1. When $\mu = 0$, the model of task-deletion mechanism returns to the Barabási model. Therefore, Fig. 4(a) displays the results of the priority queueing system of the Barabási model, and with the increasing of *p*, the relations between *P* and τ change from an exponential distribution to a power-law distribution, and these changes can be verified by the limit-analysis solutions Eqs. (13) and (15), respectively. However, in Fig. 4(b), with the increasing of *p* from 0 to 1, the waiting time distributions *P*(τ) change from an exponential distribution to a power-law distribution with an exponential cutoff, and these changes can be verified by the limit-analysis solutions Eqs. (16) and (18), respectively. Therefore, the deletion parameter μ plays a very important role in the pattern diversity of the waiting time distribution between *P* and τ .

4. Conclusions

In summary, a task-deletion mechanism is added to the priority queueing system of the Barabási model to deeper research the pattern diversity of human behaviors. The analytical solution of the model with two tasks is provided. The compact approximations are derived analytically and verified by the simulation results. With the change of task-deletion parameter, the waiting time distribution of executed tasks changes and follows different patterns, such as mixing distribution of exponential-function and power-function, exponential distribution, power-law distribution and power-function, exponential distribution, power-law distribution and power-law distribution with an exponential cutoff. The rich statistical patterns of our model are consistent with lots of empirical studies. Therefore, compared to the previous work [9], our model and analytical solutions can be used to explain theoretically the pattern diverse of human behaviors. Uncovering the mechanisms governing the timing of various human activities has significant scientific and commercial potential. Modes of human behaviors are very necessary for large-scale simulations of social organizational behaviors, ranging from modeling detailed urban to modeling financial market behaviors. Thus the current model can be extended and applied to more general situations. Our work is helpful for understanding the source of pattern diversity of human behaviors.

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